1. NO CALCULATORS ALLOWED
2. UNLESS OTHERWISE INSTRUCTED, SIMPLIFY ALL ANSWERS COMPLETELY
3. SHOW PROPER & CONCISE PRECALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS

Write $\vec{d} = <6, 8>$ as the sum of 2 vectors, one perpendicular to $\vec{g} = <3, -6>$ and one parallel to $\vec{g}$. **SCORE: _____ / 5 PTS**

$$\text{PROJ}_{\vec{g}}\, \vec{d} = \frac{\langle 6,8\rangle \cdot \langle 3,-6\rangle}{\langle 3,-6\rangle \cdot \langle 3,-6\rangle}\langle 3,-6\rangle = \frac{\overset{\text{①}}{\overline{-30}}}{\underset{\text{①}}{\underline{45}}}\langle 3,-6\rangle = -\frac{2}{3}\langle 3,-6\rangle$$

$$= \underbrace{\langle -2,4\rangle}_{\text{①}}$$

$$\langle 6,8\rangle - \langle -2,4\rangle = \underline{\langle 8,4\rangle} \text{ ①}$$

$$\underline{\langle 6,8\rangle = \langle -2,4\rangle + \langle 8,4\rangle} \text{ ①}$$ FOR THIS FORMAT/EQUATION

Let $\vec{s} = 2\sqrt{3}\vec{j} - 6\vec{i}$. **SCORE: _____ / 8 PTS**

[a] Find a vector $\vec{d}$ in the **opposite** direction as $\vec{s}$, such that $\|\vec{d}\| = 5$. Write your answer in component form.

$$-\frac{5}{\|\vec{s}\|}\vec{s} = -\frac{5}{\sqrt{12+36}}\langle -6, 2\sqrt{3}\rangle$$

$$= \underset{\left(\frac{1}{2}\right)}{\underline{-}}\, \frac{\overset{\left(\frac{1}{2}\right)}{\overline{5}}}{\underset{\text{①}}{\underline{\sqrt{48}}}}\langle -6, 2\sqrt{3}\rangle$$

$$= -\frac{5}{4\sqrt{3}}\langle -6, 2\sqrt{3}\rangle$$

$$= \underline{\left\langle \frac{5\sqrt{3}}{2}, -\frac{5}{2}\right\rangle} \text{ ①}$$

[b] If $\vec{p}$ is a vector with magnitude 6 which makes an angle of 135° with $\vec{s}$, find $\vec{s} \cdot \vec{p}$.

$$\|\vec{s}\|\|\vec{p}\|\cos\theta = \underline{4\sqrt{3}(6)\cos 135^\circ} \text{ ①}$$

$$= 4\sqrt{3}(6)\left(-\frac{\sqrt{2}}{2}\right)$$

$$= \underline{-12\sqrt{6}} \text{ ①}$$

[c] Find the direction angle of $\vec{s}$.

$$\theta_{\vec{s}} = \pi + \tan^{-1}\frac{2\sqrt{3}}{-6}$$

$$= \pi + \underbrace{\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)}_{\text{①}} = \pi - \underbrace{\frac{\pi}{6}}_{\text{①}} = \underbrace{\frac{5\pi}{6}}_{\text{①}}$$

Three forces act on an object. **SCORE: _____ / 8 PTS**

Force 1 has magnitude 12 newtons and direction angle 270°.

Force 2 has magnitude 5 newtons and direction angle 60°.

Force 3 has magnitude 8 newtons and direction angle 150°.

[a] Find the resultant of the three forces. Write your answer as a linear combination of $\vec{i}$ and $\vec{j}$.

$$12\langle \cos 270°, \sin 270° \rangle$$
$$+\,5\langle \cos 60°, \sin 60° \rangle$$
$$+\,8\langle \cos 150°, \sin 150° \rangle$$
$$= \underbrace{\langle 0, -12 \rangle}_{\frac{1}{2}} + \underbrace{\left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle}_{①} + \underbrace{\langle -4\sqrt{3}, 4 \rangle}_{①}$$
$$= \left\langle \frac{5}{2} - 4\sqrt{3}, -8 + \frac{5\sqrt{3}}{2} \right\rangle$$
$$= \underbrace{\underbrace{\left(\frac{5}{2} - 4\sqrt{3}\right)}_{\frac{1}{2}}\vec{i} + \underbrace{\left(-8 + \frac{5\sqrt{3}}{2}\right)}_{①}\vec{j}}_{①}$$

① FOR THIS FORMAT (USING $\vec{i}, \vec{j}$)

[b] The resultant of the three forces acted on the object as it moved from $(1, -4)$ to $(-3, -2)$, where all coordinates are measured in meters. Find the work done, and give appropriate units for your answer.

$$\vec{d} = \langle -3-1, -2-^{-}4 \rangle = \langle -4, 2 \rangle$$
$$\left\langle \frac{5}{2} - 4\sqrt{3}, -8 + \frac{5\sqrt{3}}{2} \right\rangle \cdot \underbrace{\langle -4, 2 \rangle}_{①} = -10 + 16\sqrt{3} - 16 + 5\sqrt{3}$$
$$= \underbrace{-26 + 21\sqrt{3}}_{①}\ \underbrace{\text{Nm OR J}}_{①}$$

① FOR EITHER ANSWER.

[FILL IN THE BLANKS] **SCORE: _____ / 7 PTS**

② IF ALL CORRECT COORDINATES

① IF 2 CORRECT COORDINATES

[a] You start at the origin in 3D, and move 12 units down, 9 units forward, and 11 units left. You are now at the point with co-ordinates $(9, -11, -12)$, you are in octant 8 ①, and you are 11 ① units away from the xz – plane.

[b] If $\vec{b} \cdot \vec{a} = -9$, then the angle between $\vec{a}$ and $\vec{b}$ is OBTUSE ①.

[c] The equation of the yz – trace of the sphere $(x+2)^2 + (y-3)^2 + (z-1)^2 = 19$ is $(y-3)^2 + (z-1)^2 = 15$ ②.

For the vectors shown below, sketch the vector $2\vec{w} - \frac{1}{2}\vec{p}$. **SCORE: _____ / 2 PTS**